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Background Noise Effects on Combustor Stability

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This paper considers the effects of background turbulent fluctuations upon a combustor's stability boundaries. Inherent turbulent fluctuations act as both additive and parametric excitation sources to acoustic waves in combustors. Although additive noise sources exert primarily quantitative effects upon combustor oscillations, parametric noise sources can exert qualitative impacts upon its dynamics; particularly of interest here is their ability to destabilize a system that is stable in the absence of these noise sources. The significance of these parametric noise sources increases with increased background noise levels and, thus, can play more of a role in realistic, high-Reynolds-number systems than experiments on simplified, lab-scale combustors might suggest. The objective of this paper is to determine whether and/or when these effects might be significant. The analysis considers the effects of fluctuations in damping rate, frequency, and combustion response. It is found that the effects of noisy damping and frequency upon the combustor's stability limits is relatively small, at least for the fluctuation intensities estimated here. The effects of a noisy combustion response, particularly of a fluctuating time delay between flow and heat-release perturbations, can be quite significant, however, in some cases for turbulence intensities as low as $u'_{\mathrm{rms}}/\bar{u} \sim$ 5–10%. These results suggest that deterministic stability models calibrated on low turbulence intensity, lab-scale combustors might not adequately describe the stability limits of realistic, highly turbulent combustors.

Nomenclature

A = pressure amplitude
a = speed of sound
C = normalizing constant, see Eq. (2)
c = covariance
D = probability density function constant, see Eq. (2)
F = background noise sources
n = interaction index, see Eq. (9)
P = probability distribution function
p = pressure
q = heat-release rate
T = temperature
T_0 = acoustic period, $2\pi/\omega_0$
t = time
u = velocity
β = defined in Eq. (11)
γ = ratio of specific heats
ζ = damping rate
σ = variance
$\tilde{\varsigma}$ = defined in Eq. (11)
τ = time delay, see Eq. (9)
Φ = defined below Eq. (22)
ϕ = pressure phase
ω = angular frequency

Subscript and superscript

$'$ = perturbation quantity
$-$ = time-averaged quantity

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I. Introduction

THIS paper describes an investigation of background noise effects upon combustion instabilities. Specifically, it addresses the effects of noise upon the operating conditions where instabilities occur. This work is motivated by the fact that combustion dynamics is a serious issue hindering the development and operation of industrial gas turbines.[1–4] These instabilities generally occur when the unsteady combustion process couples with one or more of the natural acoustic modes of the combustion chamber, resulting in self-excited oscillations that can achieve significant amplitudes. These oscillations are destructive to engine hardware and adversely affect engine performance and emissions.

Effective implementation and optimization of either passive or active methods of eliminating these oscillations requires a thorough understanding of the fundamental processes that affect the combustor's dynamics. These dynamics are controlled by a complex interplay of linear, nonlinear, and stochastic processes that affect the conditions under which instabilities occur, the amplitude of the oscillations, and the effectiveness of passive and active control approaches.

Significant effort has been expended to develop simplified, physics-based models that capture the dynamics of gas turbine combustors. For example, linear and nonlinear models of lean premixed gas turbine combustor dynamics have been proposed by Janus and Richards,[5] Dowling,[6] Lieuwen and Zinn,[7] Peracchio and Proscia,[8] and Fleifel et al.[9] Similar modeling work has been performed for other propulsion and industrial combustion systems.[10,11] Much of the early modeling work was linear[11] and attempted to determine the conditions under which a combustor would spontaneously become unstable. Although such linear analysis cannot determine the amplitude of the instabilities, they often can predict the frequency of the oscillations and, in some cases, the conditions under which instabilities occur.

The observation of "pulsed" instabilities in rockets that were linearly stable[12] and the desire to predict instability amplitudes motivated the development of models to capture nonlinear processes in combustion chambers.[13,14] Extensive work by Crocco and Cheng,[11] Zinn and Powell,[13] Culick and coworkers,[12,14] and others resulted in significant improvements in the understanding of such phenomena. A particularly significant result of this work was the demonstration that acoustic oscillations in combustion chambers could be modeled as a superposition of nonlinearly interacting oscillators. These analyses allowed for a systematic development and study of model equations describing combustor dynamics and have been used in virtually every theoretical study since.

Other work has shown that completely deterministic models, such as those just described, do not capture several important features of combustion instabilities. Rather, combustor data often exhibit stochastic features that cannot be characterized in a deterministic fashion. For example, the amplitude or phase of the limit-cycle oscillations can vary from cycle to cycle.[15] These stochastic characteristics likely occur because combustion instabilities occur in a noisy, turbulent environment that can, in some cases, cause qualitative changes in the combustor's dynamics. Analyses by Burnley,[16] Culick et al.,[17] and Clavin et al.[18] have shown that because of the presence of background noise the instability characteristics (e.g., the parameter values defining stability boundaries or the instability amplitude) are random. Thus, it is more appropriate to characterize combustor oscillations by their statistical characteristics than by a single deterministic quantity.

An important practical consequence of the presence of these stochastic processes is that they modify both the linear and nonlinear characteristics of the combustor. For example, they can affect the combustor's linear deterministic stability boundaries, that is, a combustor that is nominally stable in the absence of background noise can be unstable in its presence. Such behavior is referred to as noise-induced transitions.[19] In the same way, they can affect the combustor's nonlinear characteristics by altering the instability amplitude.[20] Also, it has been observed that actively controlled combustors exhibit dynamics that limit the extent to which oscillations can be suppressed. It has been shown that the combined effects of background noise, system time delays, and finite controller bandwidth are responsible for this behavior.[21]

Although, as just noted, background noise processes exert various quantitative and qualitative effects upon a combustor's dynamics, the objective of this paper is to specifically assess their effects upon the combustor's linear stability limits, that is, upon the operating conditions in which self-excited oscillations occur. Two prior studies are of particular relevance. Clavin et al.[18] considered the case of a noisy instability growth rate. They analyzed situations near the deterministic stability boundary, where the overall growth rate fluctuates between positive and negative values. Their analysis suggested that these fluctuations could be responsible for erratic pressure bursts sometimes observed in liquid rockets. They did not explicitly consider their effects upon the combustor's stability boundaries. Kim[22] considered the effects of spatial inhomogeneities caused by turbulent fluctuations upon wave propagation in combustion chambers. This study derives an equation for the ensemble-averaged wave field by a rigorous analysis of the governing equations, as opposed to the more phenomenological equations considered in this study and by Clavin et al.[18] Of particular interest here is is Kim's[22] analysis of the effect of random temperature fluctuations upon the system's linear stability boundary. Given the large number of assumptions made, however, it is not clear that this approach could be generalized to realistic systems.

This present work is not motivated by any particular experimental study suggesting that background noise processes appear to exert impacts upon a combustor's stability limits. It is motivated, however, by the fact that the majority of reported experimental studies and data analysis were obtained from subscale systems, which can have lower noise levels than the full-scale hardware they are emulating. As we will show next, the significance of background noise affects increases with increased background noise levels. Thus, they likely play more of a role in realistic, high-Reynolds-number systems than experiments on simplified, lab-scale combustors might suggest. Our objective here is to determine whether and/or when these effects might be significant.

II. Background

A. Effect of Parametric Excitations on System Stability

Consider first the case where an oscillator is disturbed by deterministic sources. As such, we consider the following linear, second-order oscillator equation that is similar to other reduced-order combustion dynamics models developed by a number of researchers (e.g., see Refs. 8 or 9):

$$\frac{d^2 p'(t)}{dt^2} + 2\omega_0[\varsigma + F_\varsigma(t)]\frac{dp'(t)}{dt} + \omega_0^2[1 + F_\omega(t)]p'(t) = F_A(t) \tag{1}$$

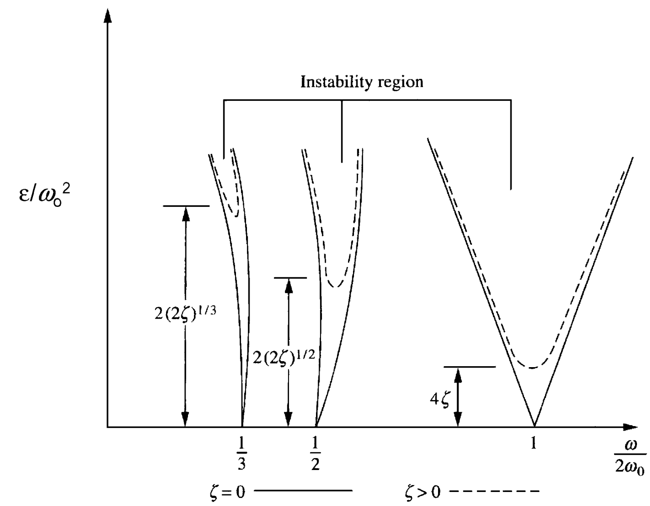


Fig. 1 Strutt diagram illustrating the stability boundaries of the Mathieu equation, Eq. (1). Adapted from Ref. 24.

The quantities $F_\zeta(t)$ and $F_\omega(t)$ denote parametric excitations of the damping and frequency, respectively, while $F_A(t)$ is an external excitation. Purely additive external sources excite oscillations in the system, even when it is damped, but do not alter its qualitative characteristics, that is, they do not affect a system's stability limits. Parametric excitations can, however, destabilize a "nominally" stable system,[23] that is, they exert qualitative impacts upon the system's dynamics. For example, consider Mathieu's equation, where the system's frequency oscillates harmonically: $F_\zeta(t) = F_A(t) = 0$ and $F_\omega(t) = \varepsilon \sin(\omega t)$. This parametric source can destabilize a nominally stable system, depending upon the frequency of oscillation ω and amplitude of disturbance ε. Figure 1 summarizes the parameter values for which Eq. (1) is stable and unstable. The figure shows that instability regions are roughly centered at frequencies of $\omega/2\omega_0 = 1/n$ (where n is a positive integer) and that the instability regime is widest at $n = 1$, that is, when the parametric excitation frequency is twice that of the system's natural frequency.[23] This destabilization arises from the fact that, in general, work is performed upon the system as its mass or stiffness are modulated. As might be expected, larger amplitudes of excitation are required to destabilize the system with increased damping.

Similar results apply for cases where the excitations are random, although the resulting analysis is considerably complicated by the fact that the disturbances have broader bandwidth and finite correlation times.

B. Experimental Evidence for Parametric Noise Effects upon Combustor Oscillations

Before proceeding, it is worthwhile to present data suggesting that these parametric disturbances exist in combustors. This is done by showing that the statistical characteristics of pressure oscillations are better captured with a model that includes parametric noise sources than one that includes only additive noise sources.

Defining the amplitude of the oscillations of the system described by Eq. (1) as $A(t)$, that is, $p'(t) = A(t)\cos[\omega_0 t + \phi(t)]$, it is shown in Ref. 23 that the stationary probability density function (PDF) of $A(t)$ is

$$P(A) = CA(DA^2 + 1)^\kappa \tag{2}$$

where the following definitions are used for the variances of the noise terms:

$$\sigma_\varsigma^2 = \langle [F_\varsigma(t)]^2 \rangle \qquad \sigma_\omega^2 = \langle [F_\omega(t)]^2 \rangle \qquad \sigma_A^2 = \frac{\langle [F_A(t)]^2 \rangle}{\omega_0^2}$$

$$D = \frac{\left(\sigma_\omega^2 + 12\sigma_\varsigma^2\right)}{4\sigma_A^2} \qquad \kappa = -\frac{4}{\pi}\frac{\varsigma + 2\sigma_\varsigma^2}{\sigma_\omega^2 + 12\sigma_\varsigma^2} \tag{3}$$

Equation (2) reduces to the Rayleigh distribution in the absence of parametric disturbances. This can be shown by taking the limit as σ_ω and $\sigma_\varsigma \to 0$:

$$P(A) = \tilde{C} A \exp\left(-\varsigma A^2 / \pi \sigma_A^2\right) \tag{4}$$

Figures 2 and 3 compare the predicted PDFs of Eqs. (2) and (3) with measured data that were obtained under stable operating conditions. These identical data were previously presented in Ref. 20 and compared with a model that only incorporated additive noise terms. These comparisons should only be thought of as curve fits, as we have simply found the parameter values that best fit the data. Although the relative ratios of the noise terms are important, their absolute magnitudes are not, as the amplitude is plotted in arbitrary units. In the curves that include parametric noise terms, we used the ratios $\sigma_A = 4\sigma_\omega = 8\sigma_\varsigma$, which give the best fit. The figures show that both models give reasonable agreement for the majority of the data, except at the more infrequently occurring, higher-amplitude values of the pressure. The model including parametric noise sources is better able to capture these excursions; this is especially illustrated in the logarithmic plot in Fig. 3. Although it is possible that nonlinear effects could also be responsible for these higher-amplitude excursions, this seems unlikely, assuming that the nonlinearities are of a "stiffening" nature. Analysis reported in Ref. 20 suggests that including nonlinear effects would reduce, rather than increase, these higher-amplitude excursions.

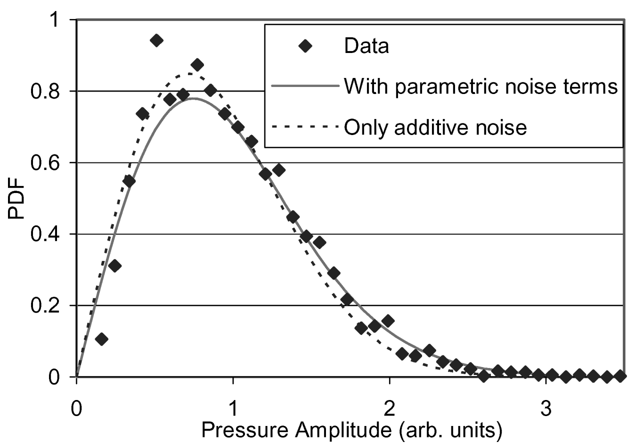


Fig. 2 Comparison of PDFs of measured pressured data and predicted distribution with and without parametric noise terms. Data obtained from Ref. 20.

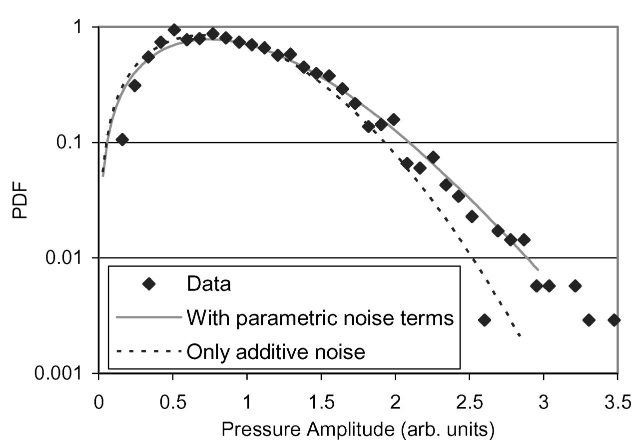


Fig. 3 Same data as in Fig. 2, except *y* axis plotted with a logarithmic scale.

These results suggest that parametric noise effects are needed to describe the measured distributions. For this particular example, it shows that purely additive noise models cannot predict the larger amplitude, but relatively infrequent, excursions of the pressure from its average values. Based upon the relative values of the respective noise process amplitudes, however, it also suggests that additive noise sources are the strongest.

III. Stability Estimates

We consider next the issue of combustor stability in the presence of parametric noise sources. The question of stability of stochastic systems is a delicate one, and different definitions for stochastic stability exist in the literature. In this paper, we use Lin and Cai's[23] definition that utilizes Lyapunov functions.

According to Lin and Cai[23] only sufficient conditions have been obtained for the stability of linear systems disturbed by nonwhite, finite variance disturbances. In this paper, we utilize the results of analysis reported in Lin's text whose method appears to have first been developed by Infante.[24] Lin and Cai[23] present the following result for the stability boundary of Eq. (1):

$$\left(1 - c_{\varsigma\omega}^2\sigma_\varsigma^2\right)\sigma_\omega^2 - 4\varsigma c_{\varsigma\omega}\left(1 - \sigma_\varsigma^2\right)\sigma_\varsigma\sigma_\omega + 4\left(1 - \sigma_\varsigma^2\right)\left((1 + \varsigma^2)\sigma_\varsigma^2 - \varsigma^2\right) = 0 \tag{5}$$

where the covariance between the parametric excitation sources is given by

$$c_{\varsigma\omega} = \frac{\langle F_\varsigma(t) F_\omega(t) \rangle}{\sigma_\varsigma \sigma_\omega} \tag{6}$$

Equation (5) shows that sufficient conditions for system stability can be derived, given knowledge of the variances and covariances between the fluctuations in damping and frequency. It should be cautioned that the stability boundaries derived from Eq. (5) are only sufficient conditions for stability. Deriving more precise conditions for regimes where instability can actually occur require specifying further information about the noise processes, such as their spectral content.

We now derive approximate descriptions of these noise variances and covariances and use Eq. (5) to assess the affects of these noise processes upon the combustor's stability.

A. Parametric Noise Variance and Covariance Estimates

Having discussed the qualitative influences that parametric noise sources can have on a system's stability, and presented data suggesting that such combustor noise sources exist, we now estimate their characteristics and resultant effects on combustor stability. Explicitly evaluating these effects upon stability requires examination of a specific combustor stability model; in this paper, we utilize a classical $n - \tau$ model that relates the response of the heat release to pressure perturbations at the flame [i.e., $q'(t) \sim np'(t - \tau)$]. We emphasize here that this is done simply to obtain explicit results in order to determine whether or when parametric noise effects should be accounted for in stability calculations; other models could equally well be utilized. We next discuss the factors that could cause fluctuations in frequency, damping, or heat release.

B. Random Heat-Release Disturbances

Here we restrict attention to the effects of parametric heat-release disturbances upon acoustic oscillations. Heat-release oscillations cause fluctuations in both damping and frequency. Recall that if the phase between pressure and heat-release oscillations is 0 or 180 deg, then damping is decreased or increased, respectively. Conversely, if the phase is around 90 or 270 deg, then the frequency is shifted without an effect upon damping. In general, the phase lies somewhere between these values so that both the frequency and damping rate are affected. Thus, assuming that the magnitude and phase relationship between heat release and acoustic disturbances are related through an interaction index n and time delay τ, it can be seen that oscillations in either will result in parametric frequency or damping disturbance that are correlated, for example, a change in time delay will cause a simultaneous frequency and damping change.

A number of processes cause fluctuations in the interaction index n. For example, equivalence ratio oscillations are known to be an important mechanism of instabilities in some combustors. It has been shown that the following expression approximately describes the amplitude of equivalence ratio oscillations at the fuel injection point, assuming that the fuel injection system is acoustically nonresponsive[7]:

$$\phi'/\bar{\phi} = -u'/\bar{u} \tag{7}$$

Thus, velocity disturbances alter the amplitude of equivalence ratio oscillations generated at fuel injection points that, in turn, affect the amplitude of the generated heat-release oscillations. Velocity oscillations can directly cause similar oscillations in the time delay, for example, by altering the convection time required for flow or mixture disturbances to reach the flame front.

We now present an approximate analysis to estimate these terms. We start with the following modified form of Eq. (1), which includes the effects of unsteady heat release[16]:

$$\frac{d^2 p'(t)}{dt^2} + 2\omega_0[\varsigma + F_\varsigma(t)]\frac{dp'(t)}{dt} + \omega_0^2[1 + F_\omega(t)]p'(t)$$

$$= (\gamma - 1)\frac{dq'(t)}{dt} \tag{8}$$

As just noted, we assume that the heat release is related to the acoustic field through the relationship

$$\frac{dq'(t)}{dt} = n\frac{dp'(t-\tau)}{dt} \tag{9}$$

The objective of the analysis is to assess the affects of fluctuations in n and τ. That is, we write $n(t) = \bar{n} + n'(t)$ and $\tau(t) = \bar{\tau} + \tau'(t)$. Assuming that the pressure oscillations are nearly harmonic with slowly varying amplitude and phase, we can approximately write these fluctuations in terms of equivalent fluctuations in damping and frequency, that is, we can approximate Eq. (8) by

$$\frac{d^2 p'(t)}{dt^2} + 2\omega_0[\tilde{\varsigma} + \tilde{F}_\varsigma(t)]\frac{dp'(t)}{dt} + \omega_0^2[1 + \tilde{F}_\omega(t)]p'(t) = 0 \tag{10}$$

where

$$\tilde{F}_\varsigma(t) = F_\varsigma(t) + F_{\varsigma n}(t) + F_{\varsigma\tau}(t) = F_\varsigma(t) - (\beta/2)[(n'/\bar{n})\cos\omega_0\bar{\tau}$$

$$+ \cos\omega_0\bar{\tau}(\cos\omega_0\tau' - 1) - \sin\omega_0\bar{\tau}\sin\omega_0\tau']$$

$$\tilde{F}_\omega(t) = F_\omega(t) + F_{\omega n}(t) + F_{\omega\tau}(t) = F_\omega(t) - \beta[(n'/\bar{n})\sin\omega_0\bar{\tau}$$

$$+ \sin\omega_0\bar{\tau}(\cos\omega_0\tau' - 1) - \cos\omega_0\bar{\tau}\sin\omega_0\tau']$$

$$\tilde{\varsigma} = \varsigma - (\beta/2)\cos\omega_0\bar{\tau} \qquad \beta = (\gamma - 1)\bar{n}/\omega_0 \tag{11}$$

where the subscripts n and τ denote the perturbation in $F_\zeta(t)$ or $F_\omega(t)$ by a noisy interaction index or time delay, respectively. Appendix A outlines the procedure used to obtain Eqs. (10) and (11) from Eqs. (8) and (9). To evaluate the stability boundaries of Eq. (10) using Eq. (5), we must determine the variances of the noise processes. The variance and covariance of $F_{\zeta n}$ and $F_{\omega n}$ are related to the variance of the interaction index, n by

$$\sigma_{\varsigma n}^2 = (\beta^2/4)\sigma_n^2\cos^2\omega_0\bar{\tau} \qquad \sigma_{\omega n}^2 = \beta^2\sigma_n^2\sin^2\omega_0\bar{\tau} \qquad c_{\varsigma\omega n} = 1 \tag{12}$$

where

$$\sigma_n^2 = \langle (n'/\bar{n})^2 \rangle \tag{13}$$

Evaluating these quantities for the time-delay fluctuations requires specifying the PDF of $\tau'/\bar{\tau}$. We assume a Gaussian PDF, although any could be used:

$$P(\tau'/\bar{\tau}) = \left(1\big/\sqrt{2\pi\sigma_\tau^2}\right)\exp\left[-(\tau'/\bar{\tau})^2\big/2\sigma_\tau^2\right] \tag{14}$$

where

$$\sigma_\tau^2 = \langle (\tau'/\bar{\tau})^2 \rangle \tag{15}$$

Evaluating the variances of interest requires calculating quantities such as $\langle \cos\omega_0\tau' \rangle$, which, utilizing Eq. (14), equals

$$\langle \cos\ \omega_0\tau' \rangle = \int P(\tau'/\bar{\tau})\cos\omega_0\tau'\ d(\tau'/\bar{\tau}) = \exp\left[-(\omega_0\bar{\tau})^2\sigma_\tau^2\big/2\right] \tag{16}$$

Evaluating similar integrals, we arrive at the following results:

$$\sigma_{\varsigma\tau}^2 = (\beta^2/4)\left\{2\cos^2\omega_0\bar{\tau}\left[1 - e^{-(\omega_0\bar{\tau})^2\sigma_\tau^2/2}\right]\right.$$

$$\left. - (\cos(2\omega_0\bar{\tau})/2)\left[1 - e^{-2(\omega_0\bar{\tau})^2\sigma_\tau^2}\right]\right\} \tag{17}$$

$$\sigma_{\omega\tau}^2 = \beta^2\left\{2\sin^2\omega_0\bar{\tau}\left[1 - e^{-(\omega_0\bar{\tau})^2\sigma_\tau^2/2}\right]\right.$$

$$\left. + (\cos(2\omega_0\bar{\tau})/2)\left[1 - e^{-2(\omega_0\bar{\tau})^2\sigma_\tau^2}\right]\right\} \tag{18}$$

$$c_{\varsigma\omega\tau} = \beta^2(\sin(2\omega_0\bar{\tau})/2)\left\{1 - 2\exp\left[-(\omega_0\bar{\tau})^2\sigma_\tau^2\big/2\right]\right.$$

$$\left. + \exp\left[-2(\omega_0\bar{\tau})^2\sigma_\tau^2\right]\right\}\big/\sigma_{\varsigma\tau}\sigma_{\omega\tau} \tag{19}$$

As can be seen, these variances and covariances exhibit a complex dependence upon σ_τ^2 and $\omega_0\bar{\tau}$. A typical result illustrated these dependencies is plotted in Fig. 4. The figure shows that $\sigma_{\omega\tau}$ and $\sigma_{\varsigma\tau}$ rise exponentially for small $\bar{\tau}/T_0$ values and oscillate about unity for larger values. The covariance $c_{\varsigma\omega\tau}$ oscillates about zero with an amplitude of unity for low $\bar{\tau}/T_0$ values and $\frac{1}{2}$ for larger values. Note that these curves are plotted for the value of $\sigma_\tau^2 = 0.01$; the dependence of these quantities upon $\bar{\tau}/T_0$ can change rather significantly for other σ_τ^2 values.

We next utilize Eq. (5) and these variance estimates to calculate the sufficient stability boundaries. Consider first the effects of a noisy interaction index n. Figure 5 plots the typical dependence of the sufficient stability regimes (denoted by shaded areas) upon $\bar{\tau}/T_0$ and σ_n. Dashed vertical lines denote deterministic stability boundaries that are independent of σ_n. These figures show that the effect of a noisy interaction index exerts primarily quantitative influences upon the sufficient stability region (e.g., the regions are slightly narrower than in the deterministic case), and the effect is quite small except for large fluctuations, that is, $\sigma_n > \sim 0.3$. For example, the stability region width is decreased by $\sim 15\%$ at $\sigma_n = 0.2$. If it is assumed that σ_n is proportional to the turbulence intensity, that is, $\sigma_n^2 \sim \langle (u'/\bar{u})^2 \rangle$,

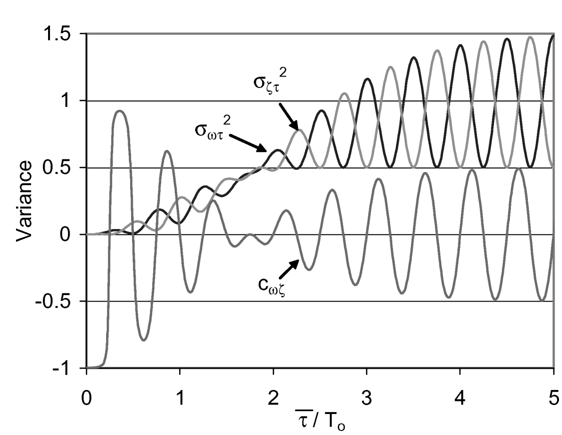


Fig. 4 Dependence of variances and covariances [calculated from Eqs. (17–19)] upon $\omega_0\bar{\tau}/2\pi = \bar{\tau}/T_0$ ($\sigma_\tau^2 = 0.01$).

then this result suggests that turbulent fluctuations with intensities below ~20% will exert minimal impacts upon the stability boundary.

Consider next the case of noisy time delay. Figure 6 plots the dependence of the sufficient stability regimes (denoted by shaded areas) upon $\bar{\tau}/T_0$ and σ_τ for several β values. These results show that a noisy time delay can cause significant qualitative changes in the combustor's stability characteristics. For example, Fig. 6a corresponds to a case where the system is nominally stable for all $\bar{\tau}/T_0$ values. In the presence of noise, however, stability is only guaranteed for relatively low σ_τ values; in some cases $\sigma_\tau < 0.05$. Because σ_τ is directly related to the turbulence intensity (e.g., for low $u'/\bar{u}$ values they are related by $\tau'/\bar{\tau} = u'/\bar{u}$ for a convected disturbance), this result implies that noisy time-delay effects can be significant for the turbulence intensities encountered in realistic systems. Note also that the potentially destabilizing effect of a noisy time delay becomes more severe at larger $\bar{\tau}/T_0$ values. This is because at a fixed σ_τ value, fluctuations in the value of $\omega\tau'$ become larger as $\bar{\tau}/T_0$ increases.

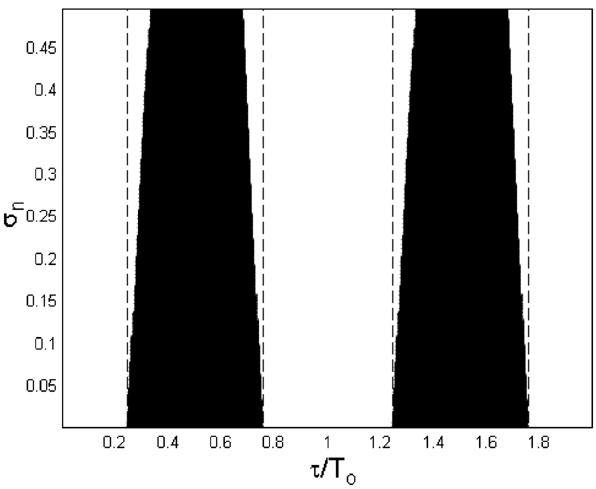


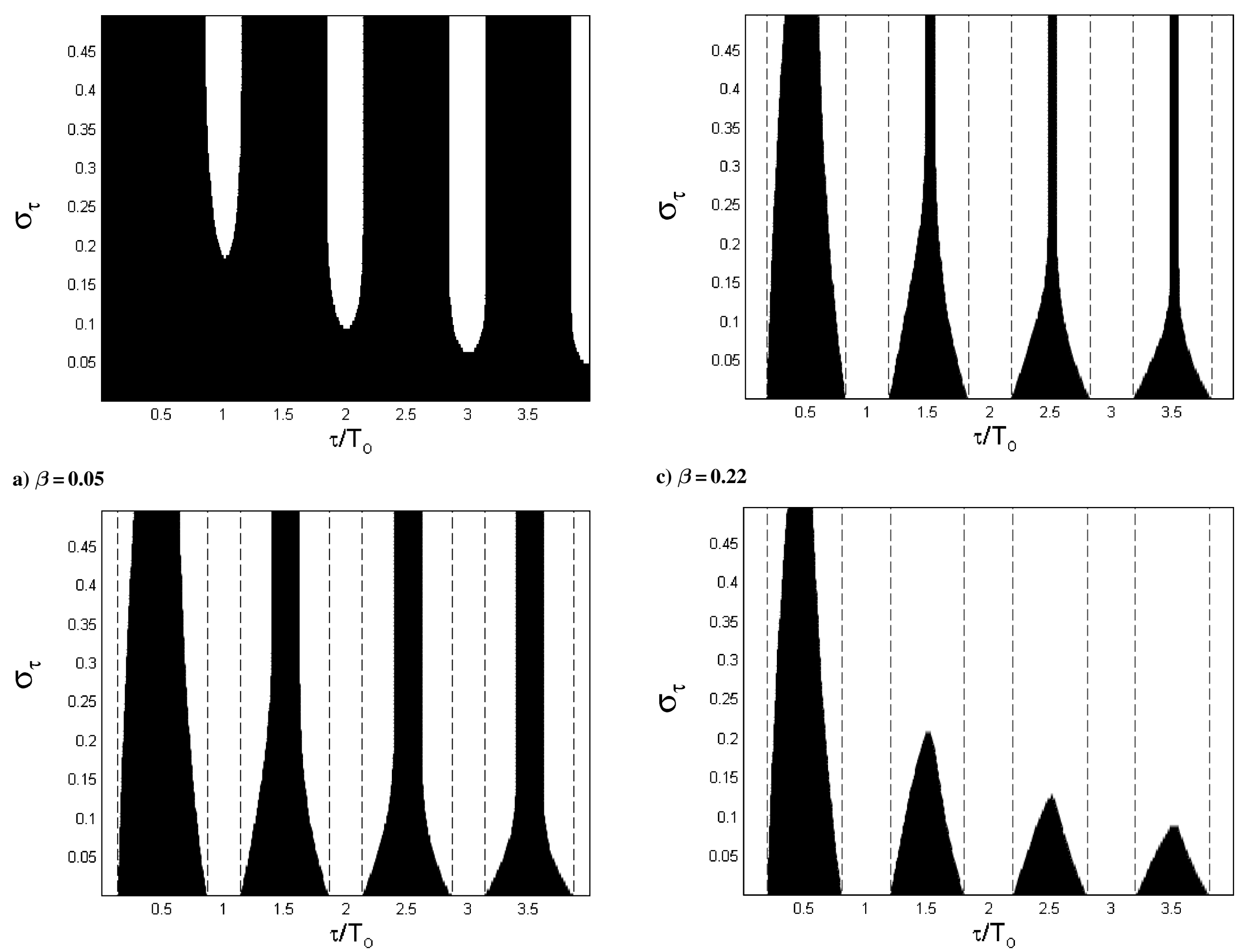
Fig. 5 Dependence of sufficient stability regimes (denoted by shaded areas) upon $\bar{\tau}/T_0$ and σ_n for case of noisy interaction index n: ---, deterministic stability boundaries ($\zeta = 0.05$ and $\beta = 0.5$).

C. Random Frequency Disturbances

The dominant factors affecting the frequency of oscillation are the chamber geometry and sound speed. The mean flow also has an effect upon frequency, but this correction has the form $1/(1-M^2)$, so that for low-Mach-number mean flows this effect should be small. Assuming that the geometry remains fixed, this leaves the sound speed, and therefore, the temperature as the dominant factor. Thus, a rough estimate for $F_\omega(t)$ is

$$F_\omega(t) = a'(t)/\bar{a} \approx T'(t)/2\bar{T} \tag{20}$$

It should be emphasized that these estimates are quite crude as they neglect the effects of combined spatial/temporal variations. Kim[22] presents a more rigorous analysis of the effects of these inhomogeneities and concludes that their effect upon the linear instability growth rate scales as $\varepsilon_t^2 \ell_c^3$, where ε_t is the magnitude of the turbulent temperature fluctuations and ℓ_c is the ratio of the turbulent correlation distance to the chamber diameter. For reference, the standard deviation of $F_\omega(t)$ can be estimated from previously reported[15]



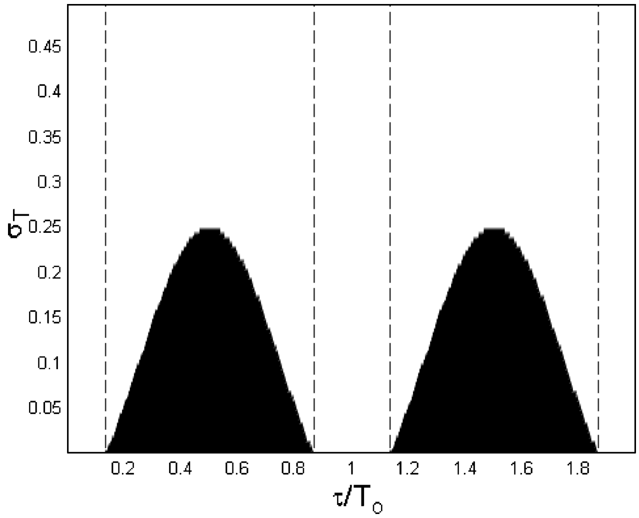
a) $\beta = 0.05$

b) $\beta = 0.15$

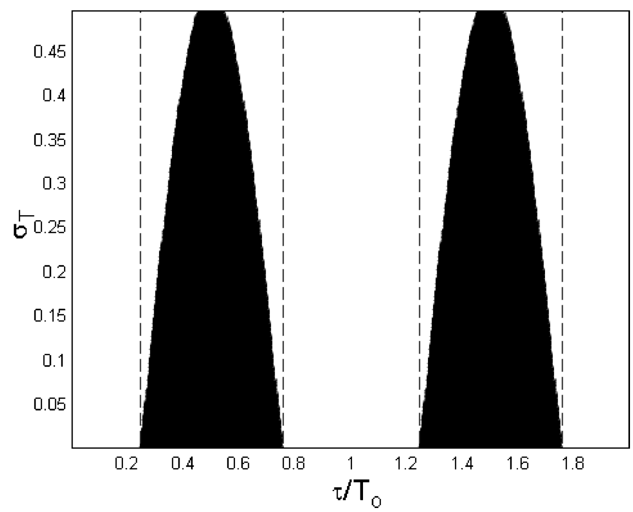
c) $\beta = 0.22$

d) $\beta = 0.3$

Fig. 6 Dependence of sufficient stability regimes (denoted by shaded areas) upon $\bar{\tau}/T_0$ and σ_t for case of noisy time delay τ: ---, deterministic stability boundaries ($\zeta = 0.05$).



a) $\beta = 0.15$



b) $\beta = 0.5$

Fig. 7 Dependence of sufficient stability regimes (denoted by shaded areas) upon $\bar{\tau}/T_0$ and σ_T (denoting the variance in temperature fluctuations) for case of noisy frequency: – – –, deterministic stability boundaries ($\zeta = 0.05$).

pressure phase disturbance measurements in a lab-scale combustor to have a value of approximately 0.01.

The effects of these fluctuations introduce quantitative differences in stability boundaries whose effect increases with decreased combustion process driving. Figure 7 plots the effect upon the sufficient stability regions, where σ_T denote the variance of the temperature fluctuations. The figure shows appreciable reductions in stability regions occur at σ_T values of greater than ~15–20%.

D. Random Damping Disturbances

Acoustic damping arises from radiation and convection of acoustic energy out of the combustor. Turbulent flow oscillations are likely to modulate the rate of convection of acoustic waves out of the combustor and, thus, exhibit an important influence on the latter damping mechanism. Damping also arises in thermal and viscous boundary layers at walls and, thus, is again likely influenced by turbulent flow fluctuations. Given these considerations, we utilized the following estimate for $F_\zeta(t)$:

$$F_\varsigma(t) = \varsigma(u'(t)/\bar{u}) \tag{21}$$

We do not present any additional stability maps showing the effect of these fluctuations because our calculations show their effect is negligible, except for turbulence intensities in excess of $u'_{\rm rms}/\bar{u} > 0.5$.

IV. Discussion and Concluding Remarks

The preceding results suggest that the effects of a noisy interaction index and frequency exert quantitative reductions in combustors stability boundaries. These effects appear to be relatively minor at turbulence intensities below ~10%. The effects of a noisy time delay appear to be most significant and can cause qualitative changes in combustor stability for turbulence intensities that can very reasonably be expected in high-Reynolds-number combustors, for example, as low as $u'_{\rm rms}/\bar{u} \sim 5\%$.

These results suggest that a more detailed look into effects of background turbulent fluctuations on combustor stability is warranted. Certainly, a more rigorous theoretical approach will provide a better understanding of the relationship between turbulent flow fluctuations, flow length scales, and the resultant fluctuations in frequency or damping. As noted earlier, some work along this line has been reported by Kim.[22] Though useful, it is unlikely that purely theoretical approaches will provide an accurate description of these noise processes. Modeling the effect of turbulent fluctuations on combustion response is difficult given the current state of the art. Also, numerous questionable assumptions are necessary in order to make analytical progress, for example, Kim's[22] analysis assumes that the acoustic time scale is much shorter than that of the turbulent flow fluctuations and that the mean flowfield is homogeneous.

A more convincing evaluation of noise effects could be obtained from relatively straightforward experiments. For example, background noise of various intensities could be excited in a lab-scale combustor, similar to the forced response experiments routinely used (usually with pulsing fuel injectors or speakers) to determine flame transfer functions or to study active instability control.[21] Such experiments could be used to determine whether and how much stability boundaries move as a function of disturbance amplitude. However, precisely quantifying the background noise characteristics will be difficult, if not impossible. In other words, it is straightforward to monitor some overall level of background pressure amplitude, but determining the more fundamental amplitudes of disturbances in frequency, damping, interaction index, and time delays would be much more difficult. Nonetheless, although data from such an experiment would necessarily be difficult to interpret, it would still provide a useful test of whether noise effects are worth considering or can safely be ignored in future studies.

Appendix: Method of Averaging

This appendix describes the procedure used to approximately equate the effects of parametric variations in heat-release parameters to fluctuations in damping and frequency. This method follows closely the method of averaging.[25] Write the pressure as $p'(t) = A(t)\cos[\omega_0 t + \phi(t)]$, and stipulate the relationship: $(\mathrm{d}A/\mathrm{d}t)\cos[\omega_0 t + \phi(t)] = A(\mathrm{d}\phi/\mathrm{d}t)\sin[\omega_0 t + \phi(t)]$. Substituting these expressions into Eq. (8) yields

$$\begin{aligned} -\frac{\mathrm{d}A(t)}{\mathrm{d}t}\omega_0 \sin\Phi(t) - A(t)\frac{\mathrm{d}\phi(t)}{\mathrm{d}t}\omega_0\cos\Phi(t) \\ = 2\omega_0^2[\zeta + F_\zeta(t)]A(t)\sin\Phi(t) - \omega_0^2 F_\omega(t)A(t)\cos\Phi(t) \\ - n\omega_0(\gamma - 1)A(t-\tau)\sin[\omega_0(t-\tau) + \phi(t-\tau)] \end{aligned} \tag{A1}$$

where $\Phi(t) = \omega_0 t + \phi(t)$. Similar to the method of averaging, we assume that $A(t)$ and $\phi(t)$ vary slowly over timescales on the order of the acoustic period and the time delay τ. In this case, we can approximate $A(t-\tau) \approx A(t)$ and $\phi(t-\tau) \approx \phi(t)$. Then, substitute $n(t) = \bar{n} + n'(t)$ and $\tau(t) = \bar{\tau} + \tau'(t)$ into Eq. (24) and expand out the terms into their constant and fluctuating components. By grouping the resulting terms, it can then be seen that the fluctuations in n and τ are equivalent to the presence of the noisy damping and frequency terms $F_\zeta(t)$ and $F_\omega(t)$ in the manner written in Eqs. (10) and (11).

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